本文取自我在北京大学博士毕业论文的致谢.

From the acknowledgements to my doctoral dissertation at Peking University.

我不是竞赛出身, 也不是所谓的北本人; 本科最后阶段做的科研, 更是和现在的方向线性无关. 可是, *"鸳鸯袖里握兵符, 将军何必是丈夫;"* 又曰: *"谪居正是君恩厚, 养拙刚于戍卒宜."* 毕竟, 谁的五年又不是五年呢?

I was not forged in maths olympiads, nor was I a Peking University-bred insider. The research I undertook in the final stage of my undergraduate years bears little relation to my current study. However, as the poem goes, *"Hidden in paired sleeves lies the token of command; must the general of an army claim command as a male birthright?"* As another classical line has it, *"Rusting in exile is surely the favour I deserve from a gracious sovereign; sheathing his dull blade is surely the wisest course for a frontier sentry saluted only by the wind."* After all, whose five years are not a testament of their own, and whose glories are merely inherited illusions?

记得第一次站在赴美访问的海关前, 我对自己说: "此去只有眼前路, 没有身后身." 我的五年博士生生涯, 甚至我的二十七年人生, 都可以被这个时刻分成明显的两截. 不必说东岩石的落日, 芝加哥的大雪; 也不必说感恩节望不尽的车流, 北岸花园的彻夜狂欢. 单是与 Brandeis 数学系里的每个人或多或少的接触, 就有无限惊喜. 我会永远记得在抵达的第一天就有贴上名字的信箱, 和在离开前的最后一天收到大半个系的联名贺卡. 我很幸运, 在价值观定型之前, 能够在这里学会如何有尊严地活着, 如何平等地尊重他人, 以及如何对身边的社区产生影响和贡献.

It is deeply etched in my mind that, standing at the U.S. customs as a visiting scholar for the first time, I said to myself that, from then on, there was no self behind me—only the road ahead. That moment became the fault line, along which my five doctoral years, as well as my twenty-seven years of life, were set apart. I need not speak of rugged East Rock at sunset or windswept Chicago deep in snow; nor need I speak of Thanksgiving traffic stretching for miles or of TD Garden roaring till dawn. Just the more-or-less communication with everyone in Brandeis math department has brought me so much joy and wonder. I shall never forget the mailbox already labelled with my full name on the very first day of my arrival, and the farewell card warmly signed by more than half of the department members on the very last day of my departure. Before my values had fully taken shape, I was so fortunate to have the opportunity to learn what it means to live with dignity, what respect we owe one another as equals, and how each of us can make impacts on and contributions to our surrounding community.

当然, 任何超脱飞扬的思想都会怦然落地的, 因为现实的引力太沉重了. 经历了这段并不平凡的五年, 我第一次意识到自己竟然也会力有未逮, 患得患失, 也会需要贵人相助才能走上人迹罕至而千差万别的道路. 为了铭记我在学术上的每一寸进步, 全部的感念应该留给以下诸位:

Of course, any thought that dared to take flight would eventually crash back to the ground, for the gravity from reality is too strong. It took those five extraordinary years to convince me that I, too, could reach the limits of my endeavor, and that I, too, could fret over gain and loss. So there is no exception that, where the road branched one hundred ways, I would need a helping hand to venture onto the one less traveled by. In order to mark down every inch of ground I have gained as a mathematician, all my gratitudes are sincerely preserved for those individuals whose names follow.

我的引路人周麒; 导师安金鹏; 科研和生活上给我无限关心的师兄关力凡, 合作者许地生; 两次出国访问的主持人 Dmitry Kleinbock, Amie Wilkinson, Danijela Damjanovic;

To my undergraduate mentor, Qi Zhou; my Ph.D. supervisor, Jinpeng An; my academic brother, Lifan Guan, and my collaborator, Disheng Xu, both of whom have been offering me unfailing support in my research and in my everyday life. To generous hosts for my two research visits abroad: Dmitry Kleinbock (Brandeis University), Amie Wilkinson (University of Chicago), and Danijela Damjanović (KTH Royal Institute of Technology).

还有多次邀请我访问的外校老师们, 包括张翼华, 石荣刚, 张润林, 杨鹏宇, 何伟鲲, 吴伟胜, 廖灵敏, 李嘉伦, 郑骋, 张涵, Anurag Rao; 以及在我自学之外知识的主要来源, 包括 Nikolay Moshchevitin, Nicolas de Saxce, Manfred Einsiedler, Elon Lindenstrauss.

To the professors who have invited me to visit their departments for several times: Yitwah Cheung, Ronggang Shi, Runlin Zhang, Pengyu Yang, Weikun He, Weisheng Wu, Lingmin Liao, Jialun Li, Cheng Zheng, Han Zhang, and Anurag Rao. And to the main sources of mathematical knowledge beyond my self-learning: Nikolay Moshchevitin, Nicolas de Saxcé, Manfred Einsiedler, and Elon Lindenstrauss.

删除这份并不太长的名单中的任何一位, 我都不会有充分的决心成为现在的自己.
Had any single name been missing from this not-too-long list, I would never have found the resolve to become who I am today.

在博士生阶段的五年, 我在学术研究中从未遇到超过一个月无进展的困难, 而最大的逆境则来自此处无可名状的非学术原因. 每当回忆起北大数学学院的旧楼新址, 我总会感慨智华楼前的草坪真干净. *"淇则有岸, 隰则有泮."* 那些让我曾经无数次夜不能寐的人和事, 终究无法压倒一条不肯气馁而死的生命, 和一颗拒绝平庸的心. 为此, 我将特别感谢以下诸位:
In all five years of my doctorate, I never went more than one month without making headway in my academic research. My most harrowing ordeals lay outside mathematics, in things I cannot describe here. Every time I think of the timeworn building and the present home of Peking University's School of Mathematical Sciences, I would always be struck by how clean the lawn was in front of Zhihua Building. As the Book of Songs has it, *"The Qi has its banks, and the marsh has its shores."* The people and events that once kept me awake through countless nights could not, in the end, extinguish a life that refused to yield to despair, or annihilate a heart that refused to settle for mediocrity. For this, my very special thanks go to those individuals I name next.

我的十四年兄弟李可, 徐时帆, 袁梓峰; 九年朋友王家文, 赵羽丰; 同行焦宇翔, 杨博寒, 魏瑜铭, Minju Lee; 舍友吴家豪, 吴润璋; Brandeis 数学系的教务兼行政人员 Martha Legace, 教学方面的教授 Merrill Keith, 和波士顿的新朋友们, 包括侯学伦, 岳翔宇, Vasiliy Neckrasov, Tariq Osman.
To Ke Li, Shifan Xu, and Zifeng Yuan, my day ones for fourteen years now, and to Jiawen Wang and Yufeng Zhao, my close friends of nine years. To my fellows in mathematics, Yuxiang Jiao, Bohan Yang, Yuming Wei, and Minju Lee, and to my roommates Jiahao Wu and Runzhang Wu. To Martha Lagace, who is solely in charge of academic and administrative affairs for the whole Brandeis mathematics department, and to Keith Merrill, a member of its teaching faculty. And to the new friends I encountered in Boston, among them Xuelun Hou, Xiangyu Yue, Vasiliy Neckrasov, and Tariq Osman.

还有安师. 正是你们的不吝善意和不求回报的支持, 使我克服了一切试图软弱或退缩的念头, 最后站到了今天这里.
And once more, to my supervisor, Professor An. It was your unstinting kindness and your selfless support that helped me resist every impulse to give in or turn back, and finally brought me to my current position.

公曰: *"天变不足畏; 祖宗不足法; 人言不足恤."* 我想起在两次自费[1]出国访问的间隙, 安师和我的对话:
As the saying goes, *"Fear no omen from Heaven. Bow to no ancestral law. Heed no idle talk."* I still remember vividly the following conversation with Professor An between my two visits abroad, both at my own personal expense.[1]

"你这么破釜沉舟, 假设将来没有达到预期, 会怎么看待年轻时的自己呢?"
"You're burning your boats," he said. "Suppose things don't turn out the way you hope. What will you think of the person you are today when you look back?"

"我只会为自己没有做过的事感到遗憾."
"I'll regret only what I haven't done."

星光殷殷, 其灿如言. 最后, 谨以此文遥望在过去的五年里错过或失去的许多人; 在他们中间, 我最怀念的还是那个面对世间五彩仍执纯白的我自己. 悠悠万事, 唯此为大: *"加强身体素质; 见证历史规律."* 知我罪我, 其惟春秋.
The stars shimmer as radiantly as the words we speak. In closing, I humbly offer these pages from afar to the many people whom I have missed or lost over those five years. Among them all, the one whom I remember most fondly is myself, who saw the world in all its colours and still held fast to pure white. Of all the things I would say to my younger self, this is the most important: *"Train your body so well that you may stay alive to see history unfold according to its own laws."* Someone may understand why I chose this road, and others may fault me for it. Ultimately, history will have the final say.

[1] 在安师的极力推动下, 第二次出国访问的机票后来得以报销.
Thanks to Professor An's unflagging persistence, I was later reimbursed for the airfare on my second visit abroad.